### At home materials
#### Year 3 Weeks 1-4 Guidance pack

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Pack 1: Reasoning with numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Session A) Counting and grouping</td>
</tr>
<tr>
<td></td>
<td>Session B) Value of the place</td>
</tr>
<tr>
<td></td>
<td>Session C) Regrouping</td>
</tr>
<tr>
<td></td>
<td>Session D) Build and adjust</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Week 2</th>
<th>Pack 2: Division and multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Session A) Describing equal groups</td>
</tr>
<tr>
<td></td>
<td>Session B) Multiplication situations</td>
</tr>
<tr>
<td></td>
<td>Session C) Arrays and area</td>
</tr>
<tr>
<td></td>
<td>Session D) Times greater</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Week 3</th>
<th>Pack 3: Addition key facts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Session A) Addition and subtraction</td>
</tr>
<tr>
<td></td>
<td>Session B) Using key facts within ten</td>
</tr>
<tr>
<td></td>
<td>Session C) Using key facts within twenty</td>
</tr>
<tr>
<td></td>
<td>Session D) Modelling problems</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Week 4</th>
<th>Pack 4: Addition and subtraction methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Session A) Addition strategies</td>
</tr>
<tr>
<td></td>
<td>Session B) Addition written method</td>
</tr>
<tr>
<td></td>
<td>Session C) Subtraction strategies</td>
</tr>
<tr>
<td></td>
<td>Session D) Subtraction written method</td>
</tr>
</tbody>
</table>
**Timing**
Each session is 30 minutes
20 minute Talk Task and 10 minute independent activity

**Session guidance**
Get them talking and grow their language.

Get them to use equipment, manipulatives, models and images to show and explain.

Challenge them to think mathematically. Use the Prompts for Thinking listed below to help them to build up habits in the way they think about mathematical situations.

**Reason it**
Explain how you know. Focus on reasons rather than answers. What could you say, do, draw or write to help someone else understand?

**Generate examples and non-examples**
What are the important features? What features are not important (e.g. colour)?

**True or false?**
If true, give examples to support your answer. If false, give a counter example.

**Find all possibilities**
Have you found all the possible answers? How do you know? Did you work systematically?

**What’s the same? What’s different?**
Compare and contrast and look for connections. How many different answers can you give?

**Always, sometimes or never true?**
Give examples to show if the statement is always, sometimes or never true. How do you know?
### Pack 1: Reasoning with number

#### Session A: Counting and grouping

**Resources needed:** Dienes thousands, hundreds, tens and ones

The purpose of this session is to get pupils talking and thinking about numbers, what they can mean and how we write them. You want to explore what pupils understand about how our number system works.

**Talk Task**

Use the images to discuss numbers and think about where and why they are used. Is it to count, measure, label, order, …? Encourage pupils to include their own examples.

Ask pupils to think about how they would try to work out how many people there are in the school (adapt to a familiar place with lots of people if this isn’t suitable for your setting). Going through the process of thinking about this will probably involve grouping the people in some way rather than thinking about each individual, e.g. there are __ teachers, there are __ children in each class,…..

Connect this experience of grouping things in order to count them with the way we write numbers. Our number system uses grouping. Discuss what they know about our number system and how it works. We use 10 digits and with them we can write any number you can think of!

Ask pupils to count from zero and show with Dienes. When they reach ten what do they do? Continue to the number 13 and ask pupils to write the number down. How can we use the Dienes blocks to show what the digits mean? The blocks allow you to see the relationship between each place in the number system. That ten ones is equal to one ten. That ten tens is equal to one hundred.

Count and build from 88 to 111. Stop every now and again and think about how to record the number you are on in digits and written words. Connect the abstract digits to the Dienes and the spoken sounds.

Count in steps of 1, 10 or 100 from different starting points both forwards and backwards.

**Activity**

The activity prompts connection of different representations of number that focus on place value understanding. Pupils complete a table showing a number with digits, words and Dienes. They then consider counting in steps of ten starting from 56.
Pack 1 Session A

**Activity:** Counting and grouping

1) Complete the table to show each number with Dienes and in words.

<table>
<thead>
<tr>
<th>number</th>
<th>Dienes</th>
<th>words</th>
</tr>
</thead>
<tbody>
<tr>
<td>154</td>
<td><img src="image1.png" alt="Dienes" /></td>
<td>One hundred and fifty four</td>
</tr>
<tr>
<td>230</td>
<td><img src="image2.png" alt="Dienes" /></td>
<td>Two hundred and thirty</td>
</tr>
<tr>
<td>307</td>
<td><img src="image3.png" alt="Dienes" /></td>
<td>Three hundred and seven</td>
</tr>
</tbody>
</table>

2) If you count in steps of 10 starting at 56, will you say these numbers? Tick the ones you will say. What other numbers would you say?

- Ninety six
- 106
- Two hundred and twenty six
- Any value greater than 56 with a 6 in the ones place

Copyright © Mathematics Mastery 2019
Pack 1: Reasoning with number

Session B: Value of the place

Resources needed: Dienes hundreds, tens and ones. Small pieces of paper.

The purpose of this task is to get pupils think about the fact that we use a place value number system: that the same digit can have a different value if it is in a different place.

Talk Task
Use three digits to explore how many different numbers can be made.

Examples are provided on the Talk Task sheet to prompt discussion. Build each number with Dienes, draw pictures of the Dienes and choose some numbers to write in words.

It would be useful to be able to write each number on small pieces of paper so they can be moved around, compared and ordered.

Explore 1-digit, 2-digit numbers and then 3-digit numbers. How might we know if we have made all the possible numbers?

Challenge pupils to think about how to explain that they have found them all. Encourage pupils to place them in order or group them by their starting (or first) digit to help convince themselves and you that there are no more options.

13, 14, 31, 34, 41, 43, 134, 143, 314, 341, 413, 431

Having made different numbers with the same digits, discuss and compare them focusing on how the value of the digit is different if it is in a different place.

In the number 134, the digit 4 has a value of 4 ones. In the number 143, the digit 4 has a value of 4 tens. In 413, the digit 4 has a value of 4 hundreds.

Sort the numbers in different ways. For example, odd and even, greater than 200 and less than 200, etc.

Extend the task by introducing a zero and exploring the options for the numbers that can be made.

Activity
The activity sheet guides students through similar experiences of using digits to write numbers and generating examples and non-examples of numbers with a given description.
Pack 1 Session B

Activity: The value of the place

1) Use these digits to create numbers for each of the properties

5 2 4

a) A number less than 100  

54, 52, 45, 42, 24, 25

b) A number greater than 300  

542, 524, 452, 425

c) An even number  

542, 524, 452, 254, 54, 52, 42, 24

d) A number that you can show with 7 Dienes blocks  

52, 25

e) An odd number  

425, 245, 45, 25

2) There are many ways to complete

<table>
<thead>
<tr>
<th>Examples</th>
<th>Non-examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number with 4 tens that is greater than 500</td>
<td>1420 3456</td>
</tr>
<tr>
<td>An even number with 3 hundreds</td>
<td>346 346</td>
</tr>
<tr>
<td>A number with 6 ones that is greater than 100 but less than 200</td>
<td>106 196</td>
</tr>
</tbody>
</table>

Copyright © Mathematics Mastery 2019
### Pack 1: Reasoning with number

### Session C: Regrouping

**Resources needed:** Dienes (at least 2 hundreds, 13 tens, 13 ones)

The purpose of this session is to explore different ways the same number can be grouped. Being able to see numbers in lots of different ways supports being able to calculate flexibly.

### Talk Task

There is a lot of information on the Talk Task sheet so fold it and look at each set of coins in turn. Discuss the relationships between the coins: that ten 1p coins is 10p, that £1 is the same as 100p or ten 10p coins.

For each set of coins, write down the value of each coin type in pence, discussing how you know and building models with Dienes to show and explain.

- 100p + 110p + 3p
- 200p + 13p
- 100p + 100p + 13p

Connect this experience to understanding our number system: that 10 ones is equal to 1 ten and that 10 tens is equal to 1 hundred.

Having looked at each set of coins, take time to look at them together and think about what is the same and what is different. Encourage pupils to think of as many different answers as they can.

The three sets of coins all have the same value of 213 pence or £2.13. None of these is the most efficient way of showing 213 pence. Ask pupils to show 213 pence with the fewest number of coins. Notice how it matches the written number with 2 hundreds, 1 ten and 3 ones.

Extend the activity by thinking about other ways the number 213 can be grouped and calculations that can be written.

### Activity

The activity sheet starts with the challenge of matching representations of three different numbers. Then pupils complete empty boxes in calculations. There are lots of patterns to find and extend within this task and you can encourage pupils to look for these. They should create more examples for each number. Extend the activity by selecting other numbers to explore.
1) Match the representations

2) Fill in the blanks to show each number in different ways. How many more can you think of?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>84</td>
<td>168</td>
</tr>
<tr>
<td>40 + 2</td>
<td>80 + 4</td>
<td>100 + 60 + 8</td>
</tr>
<tr>
<td>30 + 12</td>
<td>60 + 24</td>
<td>100 + 50 + 18</td>
</tr>
<tr>
<td>20 + 22</td>
<td>50 + 34</td>
<td>100 + 40 + 28</td>
</tr>
<tr>
<td>21 + 21</td>
<td>51 + 33</td>
<td>90 + 70 + 8</td>
</tr>
<tr>
<td>10 + 32</td>
<td>30 + 54</td>
<td>90 + 60 + 18</td>
</tr>
<tr>
<td>33 + 9</td>
<td>20 + 64</td>
<td>90 + 50 + 28</td>
</tr>
<tr>
<td></td>
<td>10 + 74</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80 + 80 + 8</td>
<td></td>
</tr>
<tr>
<td>Pack 1: Reasoning with number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Session D: Build and adjust</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resources needed: Dienes, 10 ones, 10 tens, 10 hundreds and 1 thousand</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The purpose of this session is to play with numbers and think about what can and cannot happen when you restrict or adjust.

**Talk Task**

Use exactly ten Dienes blocks to build numbers and explore the different numbers that can be shown with 10 blocks. Choose from ones, tens or hundreds and extend to thousands if appropriate.

Record the numbers and images of the numbers. There are plenty of opportunities for finding and extending patterns when generating examples.

Remember to include non-examples by discussing the numbers that cannot be shown with exactly ten blocks.

Having generated lots of examples, choose a few examples and think about what could happen if you adjust the model. Add one more block and explore possible outcomes. Take away a block and explore possible outcomes.

Draw attention to which digits change and how they change to connect to the next section.

Discuss and explore how the digits change when 10 is added to a number. Which digit will always change, which will never change and which will sometimes change? Create examples to support conclusions.

*The digit in the ones place will never change.*

*The digit in the tens place will always change.*

*The digit in the hundreds place will sometimes change.*

**Activity**

The activity sheet guides students through similar tasks of creating numbers with five Dienes blocks and thinking about how the digits change when one is added.

*The digit in the ones place always changes.*

*The digit in the tens place sometimes changes.*

*The digit in the hundreds place sometimes changes.*
Pack 1 Session D

Activity: Build and adjust

1) Draw and write numbers with **exactly five Dienes blocks**

![Dienes blocks]

If you add 1 to a number, the digit in the ones place changes.

10 + 1 = 11, 19 + 1 = 20

If you add 1 to a number, the digit in the tens place changes.

Changes: 39 + 1 = 40
Doesn’t change: 38 + 1 = 39

If you add 1 to a number, the digit in the hundreds place changes.

Changes: 199 + 1 = 200
Doesn’t change: 234 + 1 = 235

2) Circle always, sometimes or never and give examples to support your answer.

- **Always**
  - If you add 1 to a number, the digit in the ones place changes.
  - 10 + 1 = 11, 19 + 1 = 20

- **Sometimes**
  - If you add 1 to a number, the digit in the tens place changes.
  - Changes: 39 + 1 = 40
  - Doesn’t change: 38 + 1 = 39

- **Never**
  - If you add 1 to a number, the digit in the hundreds place changes.
  - Changes: 199 + 1 = 200
  - Doesn’t change: 234 + 1 = 235
# Pack 2: Multiplication and division

## Session A: Describing equal groups

**Resources needed:** Counters (or buttons, coins or other countable objects)

The purpose of this session is to explore the language and symbols of multiplication and division when describing equal groups. Groups of counters are used to show that multiplication is commutative and to explore the relationship between multiplication and division.

### Talk Task

Discuss the images of counters and ask pupils to describe what they see and notice. Look at each model in turn and ask pupils to explain how it can be used to show each of the four multiplication and division calculations below it.

Use counters to create the model and take the time to attach each number within the calculation to the model. Encourage pupils to move the counters as they explain. Example of language for first model:

- \(3 \times 4 = 12\)  
  *There are 3 groups. There are 4 counters in each group. There are 12 counters altogether.*

- \(4 \times 3 = 12\)  
  *A group of 4 counters multiplied by 3 is 12 counters*

- \(12 \div 3 = 4\)  
  *12 is split into 3 equal groups. There are 4 in each.*

- \(12 \div 4 = 3\)  
  *12 is split into groups of 4. There are 3 groups.*

Having looked at each model, look at the whole sheet and ask pupils to tell you what they know about multiplication and division. Listen and discuss what they say.

Points to include in the discussion throughout the session:

- Multiplication and division are related, there is a doing and undoing relationship. If I know a multiplication fact, I also know division facts.

- Multiplication is commutative. The order in which you multiply does not change the result. Pupils may have a fixed way of reading a multiplication calculation and you want to help them be flexible and think about \(5 \times 2\) as 2 groups of 5 or as 5 groups of 2 because these have the same value.

### Activity

The activity prompts similar experiences of describing and creating groups of counters. Then the context of seats around tables is used to apply understanding in a different situation. The final question has more than one possible answer.
Pack 2 Session A

Activity: Describing equal groups

1) Write four calculations to describe the counters

\[
\begin{align*}
4 \times 5 &= 20 \\
5 \times 4 &= 20 \\
20 \div 5 &= 20 \\
20 \div 4 &= 20 \\
\end{align*}
\]

2) Draw two different sets of counters to show the calculations

\[
\begin{align*}
3 \times 7 &= 21 \\
7 \times 3 &= 21 \\
21 \div 7 &= 3 \\
21 \div 3 &= 7 \\
\end{align*}
\]

3) Table arrangements
   a) Between 30 and 40 people are sat at tables of 4. All the tables are full. How many tables could there be?
   
   8 tables is 32 people, 9 tables is 36 people and 10 tables is 40 people

   b) Between 40 and 60 people are sat at tables of 5. All the tables are full. How many tables could there be?
   
   There could be between 8 and 12 tables

   c) I need to seat 47 people. What are my options with the tables shown above?

   10 tables of 5 with 3 spare seats
   12 tables of 4 with 3 spare seats
   9 tables of 5 and a table of 4 with 2 spare seats
Pack 2: Multiplication and division

Session B: Multiplication situations

Resources needed: Counters (or buttons, coins or other countable objects)

The purpose of this session is to explore situations involving multiplication and division and models that can be used to represent them. Building confidence with interpreting and building models will allow pupils to use them to make sense of mathematical situations and support understanding of calculation strategies.

Talk Task
The talk mat has four models and four word problems. Start by discussing the models and encourage pupils to describe what they can see. Think about the multiplication and division calculations these could represent and write them out.

Read a word problem and discuss which model can be used to represent the problem. Discuss that often there will be more than one possible model.

Spend time describing how the model represents the problem i.e. what each part of the model means in the problem. For example, *The counters can represent the sugar problem. Each counter is 1 kg of sugar. 12 kg is shared into 3 equal groups, there are 4 kg in each group.*

Discuss which calculation can be used to find a solution to each problem encouraging students to think about when there can be more than one answer. For example, a problem involving division can be thought of as multiplication with an unknown factor. *This problem is asking how many lots of 4 minutes are there in 12 minutes. I can think of this as 4 × ___ = 12*

Ask pupils to give a full sentence answer to the questions asked in each problem.

The problem involving money has the language ‘times as much’ to compare two amounts. When choosing a model, ask pupils to think about how much money there is altogether. This will draw attention to the two amounts that are being compared and so the model with two bars is a logical choice. The other models can be used and they show the amount the brother has. Times greater is the focus of session D

Activity
The activity prompts similar experiences of interpreting word problems and models. Some of the word problems have gaps to be completed. Pupils are to record calculations that are useful for finding a solution and write a full sentence answer to the question asked in the answer box.
### Activity: Multiplication situations

Complete the images models and calculations and answer the question.

<table>
<thead>
<tr>
<th>Problem:</th>
<th>Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem:</strong> How many <strong>days</strong> are there in 3 weeks?</td>
<td><img src="image" alt="Model" /></td>
</tr>
<tr>
<td>How many weeks is <strong>21</strong> days?</td>
<td></td>
</tr>
<tr>
<td><strong>Calculations:</strong></td>
<td><strong>Answer:</strong></td>
</tr>
<tr>
<td>3 × 7 = 21</td>
<td>There are 21 days in 3 weeks.</td>
</tr>
<tr>
<td>7 × 3 = 21</td>
<td></td>
</tr>
<tr>
<td>21 ÷ 3 = 7</td>
<td>21 days is 3 weeks.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem:</th>
<th>Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem:</strong> The total mass is 24 kilograms. Each weight is 4 kilograms in mass. How many weights are there?</td>
<td><img src="image" alt="Model" /></td>
</tr>
<tr>
<td><strong>Calculations:</strong></td>
<td><strong>Answer:</strong></td>
</tr>
<tr>
<td>24 ÷ 4 = 8</td>
<td>There are 6 weights.</td>
</tr>
<tr>
<td>4 × 8 = 24</td>
<td>24 kg is 6 weights with mass of 4 kg.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem:</th>
<th>Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem:</strong> 18 litres is poured into 3 buckets so that there are equal amounts in each. How much liquid is in each bucket?</td>
<td><img src="image" alt="Model" /></td>
</tr>
<tr>
<td><strong>Calculations:</strong></td>
<td><strong>Answer:</strong></td>
</tr>
<tr>
<td>6 × 3 = 18</td>
<td>There are 6 litres in each bucket.</td>
</tr>
<tr>
<td>3 × 6 = 18</td>
<td>18 litres is 3 buckets of 6 litres.</td>
</tr>
<tr>
<td>18 ÷ 3 = 6</td>
<td></td>
</tr>
</tbody>
</table>
Pack 2: Multiplication

Session C: Arrays and area of rectangles

Resources needed: Counters, 1 cm squared paper

The purpose of this session is to explore arrays and see the connection between multiplication and the area of rectangles. The area model is useful and is used in later session. You want to establish a clear connection between experiences with counters and the more abstract rectangle.

Talk Task

An array is when objects are arranged in rows and columns. Start with arrays of counters and ask: What is the same and what is different?

Discuss and write the division and multiplication calculations that the arrays of counters can represent. Review the relationship between multiplication and division and that multiplication is commutative. That the operation on two numbers can be done in either order (link with experience in session A)

Look at the rectangles with squares and rectangles without squares below the arrays of counters. Discuss what is the same and what is different?

The area of a 2-D shape is the amount of space it takes up or the size of the surface it covers. The squares are 1 cm by 1 cm and so have an area of 1 squared centimetre, 1 cm². This knowledge can be used to describe the area of the rectangles as 12 squared centimetres, 12 cm².

Connect the rectangle to the array to see that a rectangle can be used to represent multiplication. Even though you cannot count the number of squares, you can calculate the total using multiplication. Discuss how division could be shown with each model.

Use the rectangles at the end of the page to apply this understanding. Discuss how many squares each rectangle covers and how to work it out. Write the dimensions of the rectangle along each side length.

Extend the activity by providing pupils with squared paper or a geoboard and asking them to, for example, create rectangles that cover 24 squares.

Activity

The activity has three different rectangles and prompts pupils to think about how many squares each rectangle covers. They should then consider and record the multiplication and division calculations that the rectangles can represent.

Video guidance
Pack 2 Session C
Activity: Multiplication and area of rectangles

How many squares does each rectangle cover? Write calculations that each rectangle can represent.

\[
\begin{align*}
3 \times 7 &= 21 \\
7 \times 3 &= 21 \\
21 \div 7 &= 3 \\
21 \div 3 &= 7 \\
4 \times 7 &= 28 \\
7 \times 4 &= 28 \\
28 \div 7 &= 4 \\
28 \div 4 &= 7 \\
3 \times 9 &= 27 \\
9 \times 3 &= 27 \\
27 \div 9 &= 3 \\
27 \div 3 &= 9
\end{align*}
\]
**Pack 2: Multiplication**

**Session D: Multiplication to compare**

**Resources needed:** Counters (or buttons, coins or other countable objects)

**Talk Task**

The purpose of this Talk Task is to explore experiences where multiplication is used to compare two values. This involves using language such as ‘times greater’, ‘times as much’ or ‘times as many’. This can also involve using the language of fractions and pupils should be encouraged to think about how to use the phrases ‘a third of the amount’ and ‘a quarter of…’ etc.

The first activity on the talk mat shows two people with different amounts of money and speech bubbles with statements. Ask pupils to describe the image and discuss which person says which statement.

Notice the variety of language used and check pupils are comfortable with using. Move on to look at the arrows above a number line and encourage pupils to add more information to the model. Think about how to use similar language as before to describe the arrows.

- *The grey arrow is 4 times greater than the black arrow*
- *The black arrow is a quarter of the length of the grey arrow*

Encourage them to think of situations that could be represented with this model. For example, my bus journey takes 6 minutes and my friend’s journey is four times longer.

Encourage students to draw their own models and images and continue to explore the language.

For example:

*An elastic band is stretched from 3 cm to 6 cm. We can say the length is 2 times as long.*

*A stack of 4 boxes is 4 times as tall as the height of one box*

**Activity**

The activity sheet provides situations where students can apply their understanding of multiplication to compare.

The final problem challenges pupils to use multiplication to describe a situation which is not as straightforward as the others. Prompt them to compare the price and the amount.
Pack 2 Session D

**Activity:** Using multiplication to compare

1) Use multiplication to compare the amount each person has. What different sentences could each person say?

- I have 3 times more than him
- £21 is 3 times more than £7
- I can buy something that costs 3 times as much

- He has 3 times more than me
- I have a third of the amount he has
- I would need to triple my amount to have the same as him

2) An adult giraffe is 18 feet tall. It is 3 times taller than its calf.

How tall is the young giraffe?
Label the model and write a sentence
The giraffe is 6 feet tall


Use multiplication to describe this situation in as many ways as you can.

- There a 4 times more yoghurts in the 12 pack
- The 12 pack costs 3 times the price
- The 3 pack is a third of the price of the 12 pack
- The 3 pack is a quarter of the amount of yoghurts than the 12 pack
Pack 3: Addition key facts

Session A: Addition and subtraction

Resources needed: Cubes (or buttons, coins or other countable objects)

The purpose of this session is to understand the relationship between addition and subtraction. As well as to understand that addition is commutative, it can be completed in either order. Get pupils showing with cubes, saying in words and writing calculations to demonstrate their understanding.

Talk Task
Use seven cubes to show and write addition and subtraction calculations.
During this process draw out the relationship between addition and subtraction moving the cubes to show and writing calculations.
For example, if I know $5 + 2 = 7$ then I know $7 - 5 = 2$ and $7 - 2 = 5$.
Challenge understanding of the equals symbols by discussing that you can record the ‘answer’ on the left e.g. $4 = 7 - 3$ or have an operation on both sides e.g. $3 + 4 = 2 + 5$.
Include discussions about examples that involve zero and explore examples with more than two parts as prompted by the images.
Draw attention to examples that show that addition can be completed in any order e.g. $3 + 4 = 4 + 3$
Discuss that the word *commutative* is used to describe this. Look at the models and ask pupils to talk about how each one shows addition is commutative. Ask them to use the word commutative in their explanation

*This model shows that addition is commutative because…*

Ask pupils to think about the facts that this is always true, for any addition calculation. The use of letters is a way to describe this. The letters can represent any number and you can talk about them in the same way as the other models.

*Why can’t you move the numbers in a subtraction calculation in the same way?*
Ask pupils to give examples to support their explanation of this.

Activity
The activity sheet has models for pupils to interpret and describe with calculations.
The true or false task challenges the idea that the numbers in a calculation can be moved around to any position.

*Video guidance*
Pack 3 Session A
Activity: Addition and subtraction

1) Complete the calculations that each model can represent

\[
\begin{align*}
57 + 43 &= 100 \\
43 + 57 &= 100 \\
100 - 57 &= 43 \\
100 - 43 &= 57
\end{align*}
\]

\[
\begin{align*}
+4 + 7 &= 21 \\
10 + 7 + 4 &= 21 \\
21 - 7 - 4 &= 10 \\
21 - 4 - 7 &= 10
\end{align*}
\]

\[
\begin{align*}
a + b &= c \\
b + a &= c \\
c - a &= b \\
c - b &= a
\end{align*}
\]

2) Can you move the numbers around to any position? Circle the calculations that are true.

\[
\begin{align*}
3 + 4 &= 7 \\
4 + 3 &= 7 \\
7 + 3 &= 4 \\
3 + 7 &= 4 \\
4 + 7 &= 3 \\
7 + 4 &= 3 \\
3 - 4 &= 7 \\
4 - 3 &= 7 \\
7 - 3 &= 4 \\
3 - 7 &= 4 \\
4 - 7 &= 3 \\
7 - 4 &= 3
\end{align*}
\]
**Pack 3: Addition key facts**

**Session B: Key facts to 10**

**Resources needed:** Dienes ones, tens and hundreds

The purpose of this session is to explore key addition facts for all numbers up to 10 and how these can be used. The relationship between addition and subtraction should be a focus as well as extending to use with larger numbers.

**Talk Task**

A grid is provided that shows addition facts for numbers up to 10. These are all facts that can be shown with your fingers. Ask pupils to talk to you about the grid: how to read it, if they can give more information, if there are any patterns they notice.

Choose some calculations and discuss the related subtraction facts using Dienes blocks to move and show the relationship to the addition fact.

Discuss the yellow boxes in the grid and what pupils notice about these. This should involve a discussion about the number ten: how to write it and why it has two digits. Use Dienes to show that ten ones is equal to one ten.

The rest of the session focuses on how to use key facts to calculate with larger numbers.

Create the images of Dienes tens and ones to explore how $5 + 3 = 8$ can be used to complete related facts. Record addition and subtraction facts for each row of Dienes blocks in the image.

The speech bubbles show how to use a know fact to find related facts where the parts are ten times or a hundred times greater. Use Dienes or draw images of Dienes to explore addition and subtraction facts. For example,

\[
50 + 30 = 80 \quad 30 + 50 = 80 \quad 80 - 50 = 30 \quad 80 - 30 = 50
\]

Saying the name of the place the digit is in helps make a clear link to the key fact e.g. *five tens add three tens is eight tens*.

Extend the activity by repeating similar experiences with other key facts.

**Activity**

The activity sheet has addition and subtraction facts with missing numbers that show chains of related facts and then asks pupils to generate similar calculations. The robot task connects addition and subtraction with movement on a number line.

**Video guidance**
Pack 3 Session B

**Activity:** Key facts to 10

1) Complete the calculation to show how a key fact can be used:

- \(4 + 5 = 9\)  
- \(5 + 4 = 9\)
- \(14 + 5 = 19\)  
- \(50 + 40 = 90\)
- \(29 - 5 = 24\)  
- \(900 - 400 = 500\)
- \(79 - 4 = 75\)  
- \(9000 - 5000 = 4000\)

2) Write calculations that \(6 + 2 = 8\) can be used to work out.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(16 + 2 = 18)</td>
<td>(96 + 2 = 98)</td>
</tr>
<tr>
<td>(6 + 32 = 38)</td>
<td>(6 + 72 = 78)</td>
</tr>
<tr>
<td>(80 - 20 = 60)</td>
<td>(600 + 200 = 800)</td>
</tr>
</tbody>
</table>

And many more

3) This robot has two different instructions. Use A and B to move the robot from position 5 to each of these numbers. Write a calculation to describe the movement. An example is given:

- **Position 10:**  
  - Instruction: A
  - Calculation: \(5 + 5 = 10\)
- **Position 9:**  
  - Instructions: A, A, B, B
  - Calculation: \(5 + 5 + 5 - 3 - 3 = 9\)
- **Position 7:**  
  - Instructions: A, B
  - Calculation: \(5 + 5 - 3 = 7\)
- **Position 6:**  
  - Instructions: A, A, B, B, B
  - Calculation: \(5 + 5 + 5 - 3 - 3 - 3 = 6\)
- **Position 2:**  
  - Instruction: B
  - Calculation: \(5 - 3 = 2\)
- **Position 1:**  
  - Instructions: A, B, B, B
  - Calculation: \(5 + 5 - 3 - 3 - 3 = 1\)
### Pack 3: Addition key facts

#### Session C: Key facts to 20

**Resources needed:** Dienes ones, tens and hundreds

The purpose of this session is to explore key addition facts for all numbers up to 20 and how these can be used. The relationship between addition and subtraction should be a focus as well as extending to use with larger numbers.

#### Talk Task

The same grid as the previous session is provided but this time with more information. Ask pupils to discuss what has changed and repeat a similar discussion as before about more information they can give and any patterns that they notice.

Choose some calculations and discuss the related subtraction facts using Dienes blocks to move and show the relationship to the addition fact. The facts with a result greater than ten will involve regrouping and you should discuss the ‘Make 10’ strategy. For example, $7 + 5 = 7 + 3 + 2$. The 5 is partitioned, 7 and 3 make 10 and there are 2 more. Use Dienes to show the steps of this strategy, regrouping ten ones for one ten and exploring other examples.

The rest of the session focuses on how to use key facts to calculate with larger numbers.

Create the images of Dienes tens and ones to explore how $6 + 7 = 13$ can be used to complete related facts. Record addition and subtraction facts for each row of Dienes blocks in the image.

The speech bubbles show how to use a known fact to find related facts where the parts are ten times or a hundred times greater. Use Dienes or draw images of Dienes to explore addition and subtraction facts related to each. For example,

$60 + 70 = 130$  
$70 + 60 = 130$  
$130 − 70 = 60$  
$130 − 60 = 70$

Saying the name of the place the digit is in helps make a clear link to the key fact e.g. **thirteen tens subtract six tens is seven tens. Are pupils comfortable with interpreting 130 as 13 tens? Can they use Dienes to show why?**

#### Activity

The activity sheet has addition and subtraction facts with missing numbers that show chains of related facts and then asks pupils to generate similar calculations. The ‘Sum four’ task is more open and has plenty of opportunity to explore calculations.

---

**Copyright © Mathematics Mastery 2019**
Pack 3 Session C

Activity: Key facts to 20

1) Complete the calculation to show how a key fact can be used:

\[
\begin{align*}
4 + 7 &= 11 \\
9 + 5 &= 14 \\
34 + 7 &= 41 \\
50 + 90 &= 140 \\
61 - 7 &= 54 \\
1400 - 500 &= 900 \\
101 - 7 &= 94 \\
14000 - 9000 &= 5000
\end{align*}
\]

2) Write calculations that \(8 + 7 = 15\) can be used to work out.

\[
\begin{align*}
18 + 7 &= 25 \\
45 - 7 &= 38 \\
80 + 70 &= 150 \\
150 - 70 &= 80 \\
1500 - 800 &= 700
\end{align*}
\]

And many more

3) Sum four numbers.

Write different calculations you did calculations with 4 of the numbers added

Which is the largest? \[69\]  Which is the smallest? \[17\]

Write the odd numbers you can make:

\[17, 23, 51, 59, 69\]

Write the even numbers you can make:

\[28, 36, 44, 64\]

What else can you say about the numbers you can get? Multiple of 3 or 5?
**Pack 3: Addition key facts**

**Session D: Modelling problems**

**Resources needed:** Resources for drawing or building bar models

The purpose of this session is to explore problems involving addition and subtraction. The focus is on how a slight difference in language can change the structure of the problem and that being able to draw a model helps understand this structure and decide what to do.

**Talk Task**
There are six word problems all with a similar situation of two children with marbles and there are four bar models. The two questions at the bottom of the sheet do not have a bar model draw for them. Cut up the problems and bar models and you can decide if you want to include the two problems without bar models to make it more challenging or if you want to keep them separate until the end.

Read the questions and discuss the similarities. Discuss which problem can be represented by which bar model and how you know.

There are two different types of bar models, one where two parts are put together and another where two bars are compared. Connect each problem to the chosen bar model by labelling the known information and deciding what to do to work out the answer.

The last two questions can then be used to draw a model that could represent them. Some useful questions to think about:

- *What information do I know? How can I show what I know?*
- *What information am I trying to find out?*
- *How can I show the relationships between what I know and what I am trying to find out?*

**Activity**
The activity sheet provides similar experiences of engaging with addition and subtraction problems and building bar models to represent these.
Pack 3 Session D

**Activity:** Regrouping

1) Draw and label a bar model to represent each problem. Give an answer to each question.

Alicia has £6 more than Bobby. If Bobby had £10, how much do they have altogether?

**Answers**

Alicia and Bobby have £26.

Alicia has £6 more than Bobby. If Alicia had £10, how much do they have altogether?

Alicia and Bobby have £14.

Alicia has £6 more than Bobby. If they had £10 altogether, how much money does each person have?

Alicia has £8 and Bobby has £2.

2) Label the models to represent each problem and draw a model for the last question

a) Chloe is seven years younger than her sister. When she is 15, how old is her sister?

b) When her sister is 63, how old will she be?

c) How old will they both be when they have a combined age of 21?

3) Write a problem that each bar model could represent

a) A suitable problem that has two values with a difference of 7 and a total of 35

b) A suitable problem that has two values with a total of 42 where one value is 15

Copyright © Mathematics Mastery 2019
Pack 4: Addition and subtraction methods

Session A: Addition strategies

Resources needed: Scissors to cut cards, Dienes or counters

The purpose of this session is to explore addition strategies for 2-digit numbers. Encourage pupils to talk, draw and build models to explain what they are doing and discuss other strategies to complete the same or similar calculations.

Talk Task

The chosen context is packing items into crates and an image of luggage on a scale is provided to discuss situations where knowing the weight of items is important when packing or loading. Clarify that kg means kilograms.

There are cards with the weights of different items on them. These items need to be packed into crates and no crate can hold more than 100 kilograms. How many crates are needed?

Give pupils time to explore how to tackle this, moving the cards around to show the amount in each crate. This task is designed to get pupils doing lots of addition calculation and you want to encourage them to explain what they are doing and how. Ask pupils to draw or build models and write calculations to explain their strategies. Or, you can draw and write while they explain and then ask them to check if what you recorded is what they described.

Use this to explore current levels of flexibility when calculating. Do they have strategies to tackle all of the calculations they need to do? Do they change strategy depending on the numbers involved?

Some targeted questions and prompts for discussion:

Which items cannot go together in the same crate?
Is there any space left in any of the crates?

The total of all the numbers is 396 kg and they will go into four crates. This will probably involve moving them around to make them fit. A possible solution:

- 65kg and 35 kg
- 48 kg, 22 kg and 27 kg
- 53 kg, 19 kg, 18 kg and 9 kg
- 39 kg, 26 kg, 16 kg, 13 kg and 6 kg

Activity

The first question involves completing a calculation in two different ways. There is space to show the steps by recording calculations and a diagram. To complete the pyramids pupils need to use the relationship between addition and subtraction and can explore a range of subtraction strategies.

Video guidance
Pack 4 Session A

Activity: Addition strategies

1) Add these numbers using two different strategies. Draw a diagram and write calculations to show the steps of what you did.

\[ 67 + 52 + 43 \]

**Strategy 1**

\[
\begin{align*}
52 + 43 &= 95 \\
52 &\quad 92 \\
67 + 95 &= 67 + 100 - 5 \\
67 &\quad 162 \\
\end{align*}
\]

**Strategy 2**

\[
\begin{align*}
67 + 43 &= 110 \\
110 + 52 &= 162 \\
110 &\quad 52 \\
162 &\quad \\
\end{align*}
\]

2) Complete the three pyramids so that each brick is the sum of the two bricks below.

a) \[
\begin{align*}
189 \\
107 &\quad 82 \\
39 &\quad 68 &\quad 14 \\
\end{align*}
\]

b) \[
\begin{align*}
185 \\
96 &\quad 89 \\
28 &\quad 68 &\quad 21 \\
\end{align*}
\]

c) \[
\begin{align*}
347 \\
159 &\quad 188 \\
87 &\quad 72 &\quad 116 \\
49 &\quad 38 &\quad 34 &\quad 82 \\
\end{align*}
\]
**Pack 4: Addition and subtraction methods**

**Session B: Addition written method**

**Resources needed:** Dienes

The purpose of this session is to understand the written method for addition using Dienes as a tools for explaining each step. Then to explore other strategies for completing the same calculation to support development of flexibility when selecting how to calculate.

**Talk Task**

Use the deliberate errors on the sheet to prompt a discussion about how the written method for addition works. *What can you say, do and write to help the person who made the errors understand?* Focus on supporting pupils to give a clear explanation of each step of the process using Dienes blocks.

The language of regrouping is useful when explaining the errors. This word highlights that the number is grouped in a different way, the value has not changed. *5 ones add 7 ones is 12 ones. Regroup ten ones for 1 ten. Write 2 in the ones place and a small 1 near the tens place. This ten needs to be included.*

Establish what the answer is and then use the number lines, calculations and bar models to discuss different strategies to complete the same calculation. Spend time describing each diagram and how it shows the calculations below each.

The written method partitions both numbers and adds the place value parts, regrouping where necessary.

The first number line shows adding on from 45.

The second number line shows a compensation strategy where 40 is added and then 3 is subtracted because that is the same as adding 37.

The bar model shows a strategy where the parts in the calculation are adjusted. One part is three more, the other part is three less, the total is the same.

Discuss if there are any other ways that the calculation could be completed. Discuss which strategy is the most efficient. There isn’t a definite answer to this question, look for sensible reasons being given for the chosen answer.

**Activity**

The activity sheet addresses another common error of incorrectly lining up the digits, challenging pupils to correct and think about other results this error could produce. Pupils then complete calculations, using the space to show their working and thinking about other related calculations.

---

**Video guidance**

Copyright © Mathematics Mastery 2019
Pack 4 Session B

**Activity:** Addition written method

1a) Correct Sara’s error:

Sara’s error:

\[
\begin{array}{c}
3 \ 2 \ 4 \\
+\ 
4 \ 9 \\
\hline
8 \ 1 \ 4 \\
\hline
1
\end{array}
\]

Correct calculation:

\[
\begin{array}{c}
1 \ 5 \ 4 \\
+\ 
7 \ 3 \\
\hline
8 \ 8 \ 4 \\
\hline
2 \ 2 \ 7 \ \ 1
\end{array}
\]

b) If she makes the same error, what answer would she give for 324 + 49

Sara’s error:

\[
\begin{array}{c}
3 \ 2 \ 4 \\
+\ 
4 \ 9 \\
\hline
8 \ 1 \ 4 \\
\hline
1
\end{array}
\]

Correct calculation:

\[
\begin{array}{c}
3 \ 2 \ 4 \\
+\ 
4 \ 9 \\
\hline
3 \ 7 \ 3 \\
\hline
1
\end{array}
\]

2) Complete each calculation using the space to show how you did it.

a) 
\[
398 + 24 = 422
\]

\[
\begin{array}{c}
398 \\
+2 \\
\hline
400
\end{array}
\]

\[
\begin{array}{c}
2 \\
+22 \\
\hline
24
\end{array}
\]

\[
398 \ +\ 2 + 22 = 422
\]

b) 
\[
390 + 32 = 422
\]

\[
\begin{array}{c}
390 \\
+10 \\
\hline
400
\end{array}
\]

\[
\begin{array}{c}
390 \\
+22 \\
\hline
422
\end{array}
\]

\[
390 \ +\ 10 + 12 = 422
\]

c) 
\[
323 + 99 = 422
\]

\[
\begin{array}{c}
323 \\
+100 \\
\hline
423
\end{array}
\]

\[
\begin{array}{c}
+100 \\
-1 \\
\hline
100
\end{array}
\]

\[
323 \ +\ 100 - 1 = 422
\]

d) 
\[
330 + 92 = 422
\]

\[
\begin{array}{c}
330 \\
+100 \\
\hline
430
\end{array}
\]

\[
\begin{array}{c}
70 \\
+22 \\
\hline
92
\end{array}
\]

\[
330 \ +\ 100 - 8 = 422
\]

\[
330 \ +\ 70 + 22 = 422
\]

e) What other calculations have the same result?

\[
399 + 23
\]

\[
350 + 72
\]

\[
298 + 124
\]

\[
324 + 98
\]

\[
349 + 73
\]

\[
289 + 133
\]
Pack 4: Addition and subtraction methods

Session C: Subtraction written method

Resources needed: Different coloured pens to draw arrow. Money

The purpose of this session is to explore subtraction strategies for 2-digit numbers by completing the same calculations in different ways.

Talk Task
Discuss the speech bubble and ask pupils to explain what information is known about the situation and what else they could work out.

*He has £45 and he wants £72. I can work out how much more he needs to save.*

Look at each strategy for working this out. Take the time to connect the calculations to the arrows on the number lines. Asking pupils to describe how the arrows show the steps of the strategy.

Make sure that at some point you refer back to the situation to say that he needs to save £27 more to be able to buy the item.

For the last strategy, a number line is not provided. Instead money is shown and speech bubbles are shown. Act out the situation to help pupils visualise what is happening.

*I have £45.* (Place money in jar) *Now I have £50.* (Place money in jar) *Now £70.* (Place money in jar and make a clink noise) *Now £72.*

This strategy uses counting on to find the difference between 45 and 72. Ask pupils to draw a number line to show this strategy. The line will be different to the ones on the sheet because it shows that 27 is the distance between 45 and 72 rather than 45 is the distance between 27 and 72. As a result, you may need to repeat the actions above to help pupils think about what they are trying to show. Encourage them to think about where they are starting, what arrows to draw, in what direction and what labels to write.

Extend the activity by creating a similar experiences with a situation involving another subtraction calculation.

Activity
The activity sheet provides four descriptions of ways to complete the same calculation. For each, pupils are to draw arrows on a number line to show the strategy. Then they choose two different strategies and show the steps by writing calculations and drawing a diagram.
1) Draw arrows on each diagram to show the strategy that is described. For example, for $63 - 48$:

- Subtract 50 then add 2.
- Subtract 3 then subtract 40 then subtract 5.
- It is the same as 65 subtract 50.
- Find the difference between 48 and 63.

2) Calculate $135 - 98$ in two different ways and draw a diagram to show each. There are many possible strategies and diagrams.

**Strategy 1**

$135 - 100 + 2$

**Strategy 2**

Find the difference between 98 and 135.
### Pack 4: Addition and subtraction methods

#### Session D: Subtraction written method

**Resources needed:** Dienes

The purpose of this session is to understand the written method for subtraction, using Dienes as a tool for explaining each step. There is a focus on the role of regrouping and identifying when and where it will happen.

**Talk Task**

Use the deliberate errors on the sheet to prompt a discussion about how the written method for addition works. *What can you say, do and write to help the person who made the error understand?* Focus on supporting pupils to give a clear explanation of each step of the process using Dienes blocks.

The word ‘regroup’ is useful: *5 ones is greater than 2 ones so I need to regroup. Regroup 1 ten for 10 ones. Cross out 8 and write 7 and write 1 beside the 2.* Have pupils point to these as they explain regrouping. *8 tens and 2 ones is the same as 7 tens and 12 ones. 12 ones subtract 5 ones is 7 ones.*

Repeat with another example, asking pupils to think of an error that could be made for 76 − 48 and clearly explaining how to complete correctly.

For the next section, challenge pupils to generate examples that:
- do not involve regrouping
- involve regrouping from the tens to the ones
- involve regrouping from the hundreds to the tens
- involve regrouping from the hundreds and from the tens

Ask: *What digits can go in each box so that the calculation…?*

Success with the subtraction algorithm needs secure understanding that a number can be grouped in different ways. During this task focus on the way that 243 is grouped for each example generated. Use the images of Dienes and the calculations to discuss when each way of grouping is needed and why.

Extend the activity by saying a number to subtract from 243 and pupils point to the representation of Dienes that is needed.

**Activity**

The activity sheet provides similar experiences where pupils demonstrate understanding of the written method for subtraction. Extend the activity by selecting calculation that could be completed with a more efficient strategy.
1) Work out the missing digits to correctly complete each calculation.

a) 
\[
\begin{array}{ll}
5 & 3 \\
- & 3 & 6 \\
\hline
1 & 7 \\
\end{array}
\]

b) 
\[
\begin{array}{ll}
5 & 1 & 6 \\
- & 2 & 2 & 4 \\
\hline
2 & 9 & 2 \\
\end{array}
\]

c) 
\[
\begin{array}{ll}
3 & 8 & 4 \\
- & 3 & 2 & 6 \\
\hline
5 & 8 \\
\end{array}
\]

2) Choose numbers that will create calculations that will need

a) regrouping once

\[
\begin{array}{ll}
4 & 8 & 5 \\
- & & \\
\hline
357, 291, 89, ...
\end{array}
\]

b) regrouping twice

\[
\begin{array}{ll}
4 & 8 & 5 \\
- & & \\
\hline
397, 298, 96, ...
\end{array}
\]

3) Write subtraction calculations with the answer 167 and sort them:

**do not involve regrouping**

- \(298 - 131 = 167\)
- \(189 - 22 = 167\)
- \(478 - 311 = 167\)

**involve regrouping**

- \(248 - 59 = 167\)
- \(184 - 17 = 167\)
- \(413 - 246 = 167\)
Loved a session?
Got some ideas for improvements?
Spotted a typo?

Let us know your feedback [here](#)